Theoretical and Experimental K^++ Nucleus Total and Reaction Cross Sections from the KDP-RIA Model

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Abstract

The 5-dimensional spin-0 form of the Kemmer-Duffin-Petiau (KDP) equation is used to calculate scattering observables [elastic differential cross sections ($d\sigma/d\Omega$), total cross sections (σ_{Tot}), and reaction cross sections (σ_{Reac})] and to deduce σ_{Tot} and σ_{Reac} from transmission data for K^++ ^{6}Li, ^{12}C, ^{28}Si, and ^{40}Ca at several momenta in the range $488 - 714$ MeV/c. Realistic uncertainties are generated for the theoretical predictions. These errors, mainly due to uncertainties associated with the elementary K^++ nucleon amplitudes, are large, so that the disagreement that has been noted between experimental and theoretical σ_{Tot} and σ_{Reac} is not surprising. The results suggest that the K^++ nucleon amplitudes need to be much better determined before unconventional medium effects are invoked to explain the data.

24.10Jv,25.80Nv

I. INTRODUCTION

For K^+ mesons of momenta $500-1000$ MeV/c (laboratory), a simple first-order impulse approximation model should account for the main features of K^++ nucleus (A) scattering observables. Such expectation arises from the fact that the K^++ nucleon (K^+N) effective interaction is relatively weak, hence multiple scattering corrections to the first-order impulse approximation predictions should be relatively small [1]. Thus it was surprising that the first 800 MeV/c elastic scattering differential cross section data [2] for ^{12}C(K^+,K^+) and ^{40}Ca(K^+,K^+) were consistently underestimated by a number of different first-order impulse approximation calculations [3–5]. In addition, calculated total cross sections for $K^+ + A$ were found to be much smaller [5,6] than experimental values [7,8]. These findings prompted suggestions that unconventional medium effects might explain the discrepancies [4,9–11].

The disagreement between the calculated elastic differential cross sections and the data [2] does not provide firm evidence for medium effects because of the 17% absolute normalization uncertainty for the data; this alone can account for much of the discrepancy. Indeed, more recently, it was shown that 715 MeV/c elastic differential cross section data for ^{12}C(K^+,K^+) are well-described by first-order impulse approximation calculations [12]. Yet these calculations [6] did not fit the total cross section data for K^++^{12}C at similar energies. Friedman *et al.* [13] noted, however, that the experimental total cross sections [7,8] are, in fact, model-dependent quantities, and that it is essential to use the same $K^+ + A$ scattering model for obtaining the "experimental" total cross sections from measured transmission data as is used for calculating theoretical total cross sections. They reanalyzed data from a transmission experiment and explored the model-dependence of the deduced total (σ_{Tot}) and reaction (σ_{Reac}) cross sections. In spite of the fact that care was taken to conduct a self-consistent analysis of the data, the authors of Ref. [13] concluded that "there seems to remain a significant and puzzling discrepancy between theory and experiment for K^+ nuclear interactions at intermediate energies ($p_L \approx 500-800$ MeV/c)."

Before new and unconventional physics explanations are given for discrepancies between

experiment and theory, it is important to explore all theoretical uncertainties so that realistic "error bars" are associated with the theoretical predictions. In this work we present the results of a study in which the 5-dimensional spin-0 form of the Kemmer-Duffin-Petiau (KDP) equation [3] is used to calculate $K^+ + A$ scattering observables and to deduce total and reaction cross sections from transmission data. The KDP equation resembles the Dirac equation in form, and the meson-nucleus optical potential is constructed in a manner similar to that used to generate the relativistic impulse approximation (RIA) [14] optical potential. The meson-nucleus optical potential in the KDP-RIA approach consists of large and nearly cancelling scalar and vector (time-like) components which are determined by folding the elementary K^+N amplitudes [15] with the relativistic mean-field Hartree densities of Furnstahl *et al.* [16]. The calculated scattering observables are thus subject to the $\pm$ 15% uncertainty in the elementary amplitudes [17] and to uncertainties in the nuclear densities.

The KDP-RIA model was used to calculate the $K^+ + A$ total ($\sigma_{\rm Tot}$) and reaction ($\sigma_{\rm Reac}$) cross sections for K^++ ^{6}Li, ^{12}C, ^{28}Si, and ^{40}Ca at several momenta in the range $488 - 714$ MeV/c; the same model was used to extract experimental $\sigma_{\rm Tot}$ and $\sigma_{\rm Reac}$ from transmission data. We also calculated the 715 MeV/c K^++^{12}C elastic differential cross section ($d\sigma/d\Omega$) for comparison with data.

As discussed in the following, our results for the deduced $\sigma_{\rm Tot}$ and $\sigma_{\rm Reac}$ are in basic agreement with those of Ref. [13]. But for the first time, the effects of uncertainties in the elementary K^+N amplitudes on the theoretical $\sigma_{\rm Tot}$, $\sigma_{\rm Reac}$ and $d\sigma/d\Omega$ (for 715 MeV/c K^++^{12}C) are calculated. The error bands associated with these theoretical predictions are large, so that the disagreement between experimental and theoretical $\sigma_{\rm Tot}$ and $\sigma_{\rm Reac}$ is not necessarily indicative of new, unconventional physics.

II. DEDUCING TOTAL AND REACTION CROSS SECTIONS FROM TRANSMISSION DATA

Transmission cross section experiments such as those of Refs. [7,8,13] use transmission arrays which consist of a series of thin cylindrical counters of increasing radii whose axes coincide with the beam axis. Thus, measurements summing the $\geq i^{th}$ counters determine a transmission cross section $\sigma_{\rm Trans}(\Omega_i)$ for scattering out of a solid angle Ω_i. For uncharged particles $\sigma_{\rm Trans}(\Omega_i)$ is a well-behaved function near $\Omega_i = 0$, and the total cross section is found by measuring $\sigma_{\rm Trans}(\Omega_i)$ for several values of Ω_i near zero and then extrapolating $\sigma_{\rm Trans}(\Omega_i)$ to $\Omega_i = 0$.

For K^+ or other charged particles, $\sigma_{\rm Trans}(\Omega_i)$ is not well-behaved near $\Omega_i = 0$ since the Coulomb interaction leads to an infinite total cross section. However, a finite total nuclear cross section ($\sigma_{\rm Tot}$) can be determined if Coulomb effects are removed. Thus, for each measured transmission cross section, appropriate Coulomb correction terms are subtracted. The corrected partial cross sections are then fit to a polynomial in Ω_i and, by extrapolating the fit to $\Omega_i = 0$, the finite quantity $\sigma_{\rm Ext}$ is determined:

$$\sigma_{\rm Ext} = \lim_{\Omega_i \to 0} \left[\sigma_{\rm Trans}(\Omega_i) - \text{calculated corrections}\right]. \tag{1}$$

The final value of the total cross section, $\sigma_{\rm Tot}$, is given by

$$\sigma_{\rm Tot} = \sigma_{\rm Ext} - \sigma_K - \sigma_{\pi-\mu} - \sigma_{\rm A^t} \tag{2}$$

where σ_K and $\sigma_{\pi-\mu}$ correct for kaons which decay between the target and detector and for the pion and muon contamination from these decays, and the $\sigma_{\rm A^t}$ term corrects for target impurities [7,8]. While Eq. (2) concerns experimental corrections, some model of the $K^+ + A$ interaction must be used to calculate the correction terms in Eq. (1). Thus, $\sigma_{\rm Ext}$ and $\sigma_{\rm Tot}$ are model-dependent quantities. At a minimum, when comparing experimental and theoretical total cross sections, the *same* model should be used to calculate the theoretical total cross sections as is used to calculate the correction terms used to remove Coulomb effects.

The necessary correction terms are found using the method of Ref. [18]. The scattering amplitude f, found using an optical model for the interaction, is split into a Coulomb distorted nuclear part, f_N, and a Coulomb part, f_C, by adding and subtracting the Coulomb amplitude,

$$\begin{aligned} f &= (f - f_C) + f_C \\ &= f_N + f_C, \end{aligned} \tag{3}$$

where the Coulomb distorted nuclear amplitude (f_N) is defined in the last equation. The elastic differential cross section is written as the sum of three terms:

$$\frac{d\sigma}{d\Omega} = |f|^2 = |f_N|^2 + |f_C|^2 + 2\mathrm{Re}\, f_N f_C^*. \tag{4}$$

The following quantities are defined for a given solid angle Ω_i:

$$\sigma_C(> \Omega_i) = \int_{\Omega_i}^{4\pi} d\Omega |f_C|^2, \tag{5a}$$

$$\sigma_{CN}(> \Omega_i) = 2\mathrm{Re} \int_{\Omega_i}^{4\pi} d\Omega f_N f_C^*, \tag{5b}$$

$$\sigma_e(< \Omega_i) = \int_0^{\Omega_i} d\Omega |f_N|^2. \tag{5c}$$

Using these definitions, Eq. (1) becomes

$$\sigma_{\mathrm{Ext}} = \lim_{\Omega_i \to 0} \left[\sigma_{\mathrm{Trans}}(\Omega_i) - \sigma_C(> \Omega_i) - \sigma_{CN}(> \Omega_i) + \sigma_e(< \Omega_i) + \sigma_I(< \Omega_i)\right], \tag{6}$$

where the inelastic term $\sigma_I(< \Omega_i)$, assumed to be small, is neglected in obtaining the limit. For this model, the theoretical total cross section is found by using a partial wave expansion of the scattering amplitude. The expression is given in Eq. (20) of Ref. [18].

Determination of the reaction cross section follows a similar procedure. As outlined in Ref. [13], the reaction cross section is defined to be the integral cross section for removal of particles from the elastic channel. In terms of the measured transmission cross sections for scattering out of a solid angle Ω,

$$\sigma_{\text{Trans}}(\Omega) = \sigma_{\text{Reac}} + \int_{\Omega}^{4\pi} d\Omega \left(\frac{d\sigma}{d\Omega}\right)_{\text{elastic}} - \int_{0}^{\Omega} d\Omega \left(\frac{d\sigma}{d\Omega}\right)_{\text{inelastic}} . \tag{7}$$

Since the small, inelastic term vanishes as $\Omega \to 0$, the experimental total reaction cross section is found by extrapolating the quantity

$$\sigma_{\text{Reac}}(\Omega) \equiv \sigma_{\text{Trans}}(\Omega) - \int_{\Omega}^{4\pi} d\Omega \left(\frac{d\sigma}{d\Omega}\right)_{\text{elastic}} \tag{8}$$

to $\Omega = 0$ and subtracting the σ_K, $\sigma_{\pi-\mu}$, and $\sigma_{\text{A}^{\text{t}}}$ experimental corrections.

III. RESULTS, DISCUSSION, AND CONCLUSIONS

The KDP-RIA model was used to calculate scattering observables for $450 - 750$ MeV/c $K^+ + {}^6$Li, ^{12}C, ^{28}Si, and ^{40}Ca. The same model was then used to extract experimental σ_{Tot} and σ_{Reac} from transmission data [7,8,13,19] spanning $488 - 714$ MeV/c. Figs. 1–4 show (solid circles) the experimental σ_{Tot} and σ_{Reac} cross sections obtained here. Also shown in Figs. 1–4 (solid squares) are experimental values which we obtained from the transmission data [7,8,13,19] using model-dependent corrections derived from solution of the Schrödinger equation with relativistic kinematics and the "$t\rho$" optical potential from Ref. [13]. The error bars are statistical only. Our "$t\rho$" cross sections are consistent with those in Table II of Ref. [13]. As seen from Figs. 1–4, the model-dependences in the experimental cross sections are, in general, larger than the statistical errors and are greater for σ_{Tot} than for σ_{Reac}.

The predicted total and reaction cross sections from the KDP-RIA theoretical model are shown as shaded bands in Figs. 1–4. The bands result from the $\pm$ 15% uncertainty in the elementary K^+N amplitudes [17]. Contributions to the error bands due to uncertainties in the nuclear densities were studied in Ref. [13] and shown to be small and were not included here. Some of the conventional $K^+ + A$ medium corrections have been shown to contribute only a few percent to the first-order impulse approximation predictions (Ref. [13] and references therein) and were not included here. Additional, but conventional medium

corrections (*e.g.* effects due to Pauli blocking and nuclear binding potentials in intermediate K^+N scattering states) and second-order correlation terms also remain to be included.

In Fig. 1 of Ref. [13] the optical model contributions to $\sigma_{\rm Reac}$ are shown to be less than that for $\sigma_{\rm Tot}$ and to vanish as Ω increases. This suggests that $\sigma_{\rm Reac}$ is the more reliable quantity (*i.e.,* less model-dependent) that may be derived from transmission measurements. In viewing the uncertainty bands in Figs. 1–4, it is seen that the predicted reaction cross sections are indeed less sensitive to uncertainties in the input. Given the uncertainties in the theoretical predictions, the agreement with the ^{6}Li data is reasonable, whereas the predictions for the heavier targets are systematically smaller than the data, and may suggest that some additional dynamics in the K^+-nucleus interaction remains to be taken into account. However, the mass-dependence may also indicate a still unrealized experimental problem associated with Coulomb scattering corrections owing to the strong Z^2-dependence of Coulomb scattering.

In Table I we compare the results of the present work (last row) for experimental $\sigma_{\rm Reac}$ and $\sigma_{\rm Tot}$ with those (first and second rows) taken from Table IV of Ref. [13]. The $t\rho$ potential of Ref. [13] is proportional to the product of the forward K^+N scattering amplitude $[f_{c.m.}(0)]$ and the nuclear density $\rho(r)$, while the DD potential of Ref. [13] is an *ad hoc* phenomenological density-dependent modification of the interaction to constrain the analysis to fit elastic scattering data. The $DD - t\rho$ comparison shows that the experimental $\sigma_{\rm Reac}$ is not sensitive to the choice of potential, while the same cannot be said for $\sigma_{\rm Tot}$, where the differences span $5 - 11\%$.

In Fig. 5 the KDP-RIA prediction for the 715 MeV/c K^++^{12}C elastic differential cross section is compared with the data of Ref. [12]. The shaded band indicates the uncertainty due to the $\pm$ 15% uncertainty in the K^+N amplitudes. The agreement with the data is good, but the shaded error band in this figure, as well as those in Figs. 1–4, suggest that the elementary K^+N amplitudes need to be better determined if progress is going to be made. The present situation is similar to that encountered during the early days of medium energy p+A studies [20] when the elementary p+N amplitudes were not sufficiently well-determined

at the momentum transfers important for generating $p + A$ optical potentials.

In conclusion, it is imperative that the same model be used to obtain the total and reaction cross sections from the measured transmission cross section data as is used to make theoretical predictions for comparison. To enable consistent analyses by others in the community, the transmission cross section data should be included in publications which present total and reaction cross sections extracted from such data.

Recent $K^+ + A$ transmission cross section data were analyzed using the relativistic KDP-RIA model. The model-dependence in the deduced, experimental total and reaction cross sections was discussed, and the uncertainties in the corresponding theoretical predictions owing to uncertainties in the elementary K^+N amplitudes were calculated. Our experimental σ_{Tot} and σ_{Reac} cross sections are consistent with those found in Ref. [13]. Although the theoretical predictions underestimate the experimental quantities, improved knowledge of the K^+N amplitudes is required before studies of possible K^+-nucleus medium effects can meaningfully be pursued.

The authors thank Professor E. D. Cooper, University College of the Fraser Valley, for his many contributions to this work. The authors also thank Dr. Robert Chrien of Brookhaven National Laboratory, Professor Reyad Sawafta of North Carolina A&T State University, and Dr. Ruth Weiss and Professor Eli Piasetzky of the Tel-Aviv University for their helpful discussions and information regarding the experimental aspects of $K^+ + A$ total cross section analysis. B.C. Clark, L. Kurth Kerr and S. Hama acknowledge the hospitality of the National Institute for Nuclear Theory, University of Washington where part of this work was done. This work was supported in part by NSF PHY-9511923, the U. S. Department of Energy Grant DE-FG03-94ER40845, and The Robert A. Welch Foundation Grant F-604.

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TABLES

TABLE I. $K^+ + A$ total and reaction cross sections extracted from 714 MeV/c transmission data using three different models for the extrapolations.

	Reaction (mb)				Total (mb)			
Potential	^{6}Li	^{12}C	^{28}Si	^{40}Ca	^{6}Li	^{12}C	^{28}Si	^{40}Ca
DD [a]	80.0	149.2	317.7	413.4	91.2	192.1	433.9	589.6
tρ [a]	79.3	149.3	317.5	412.9	87.0	175.6	396.5	528.4
KDP-RIA [b]	81.2	151.9	316.9	413.9	88.9	180.4	405.7	547.1

[a] From Ref. [13].

[b] Using the same extrapolation method as Ref. [13].

FIGURES

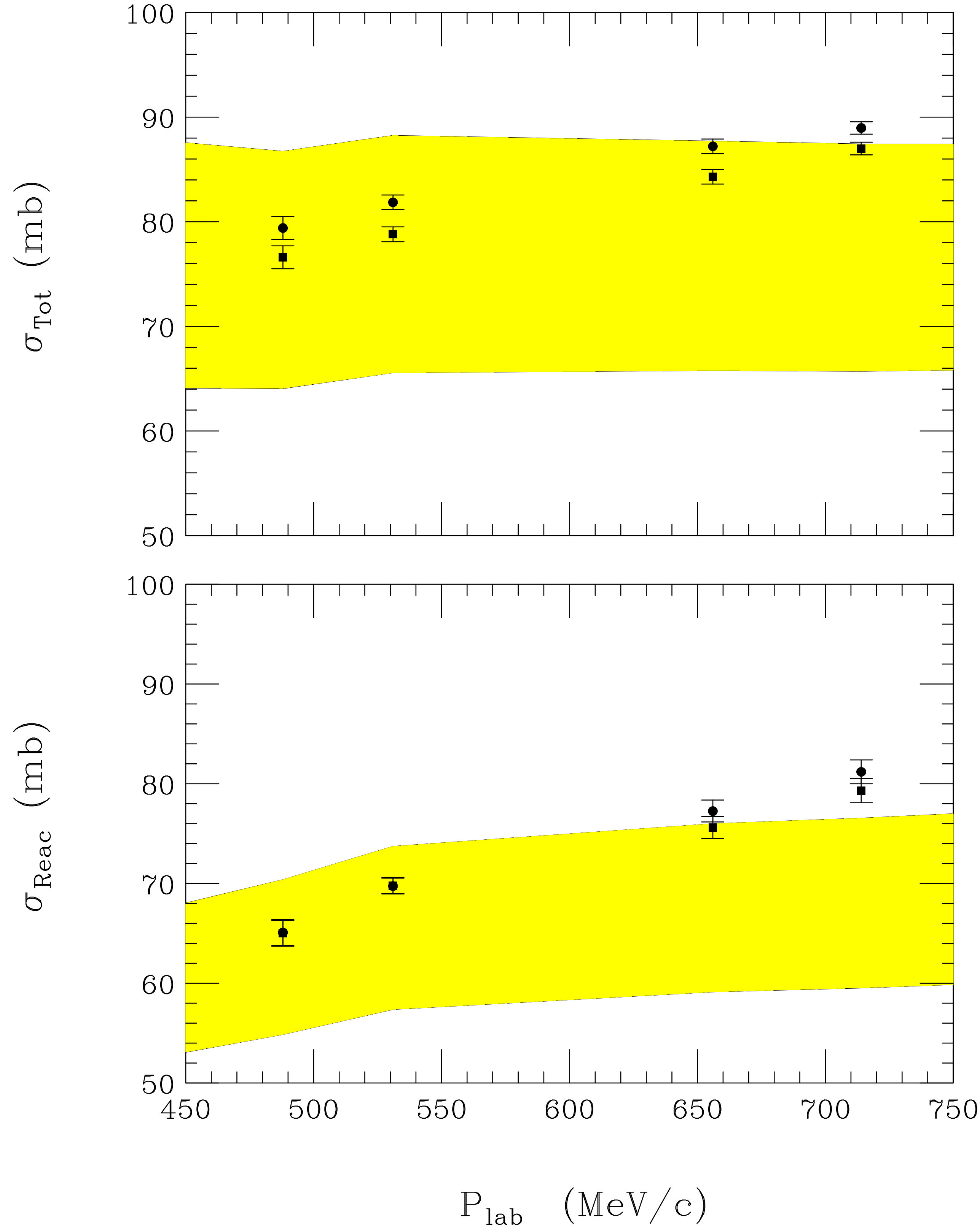
FIG. 1. The experimental and theoretical total cross sections and reaction cross sections for $K^{+}+{}^{6}$Li as a function of incident laboratory momentum. The experimental values obtained using the KDP-RIA relativistic optical model calculated corrections are shown as solid circles and those obtained using the optical model of Ref. 13 are shown as solid squares. The theoretical total and reaction cross section results are plotted as a band of values which take into account the $\pm$ 15% uncertainty in the elementary $K^{+}N$ amplitudes used in the calculation.

FIG. 2. Same as Fig. 1 except for $K^{+}+{}^{12}$C.

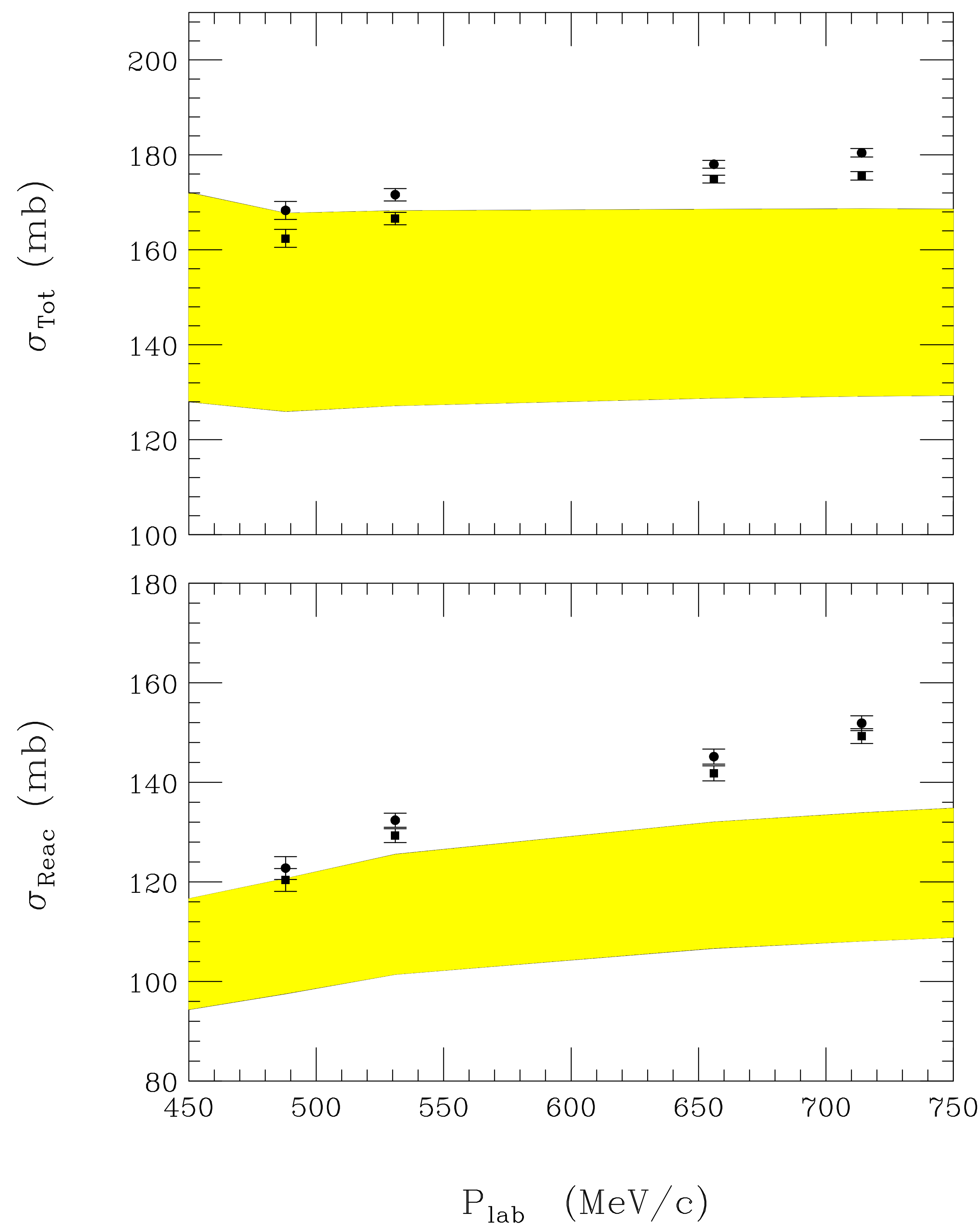
FIG. 3. Same as Fig. 1 except for $K^{+}+{}^{28}$Si.

FIG. 4. Same as Fig. 1 except for $K^{+}+{}^{40}$Ca.

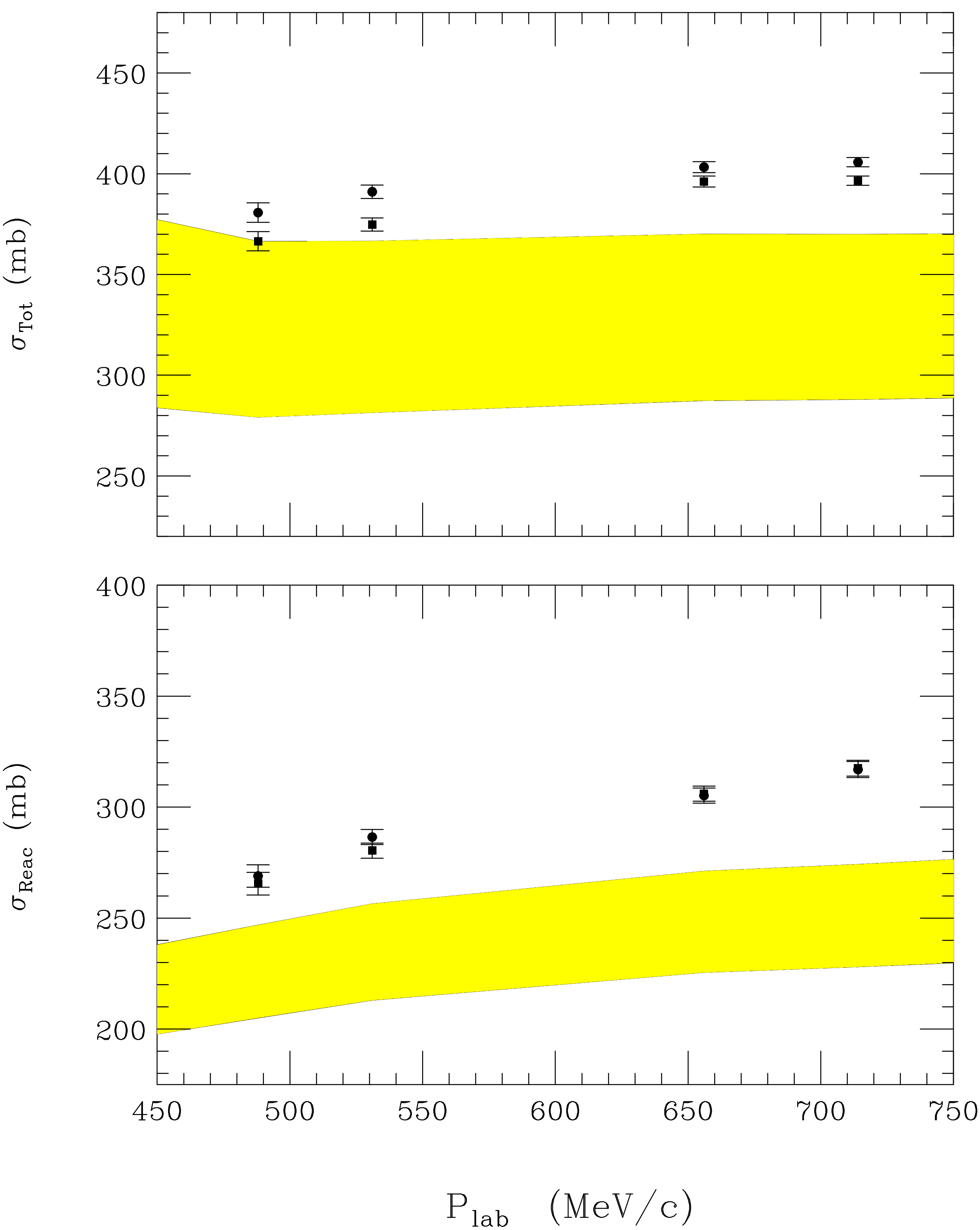
FIG. 5. The experimental and theoretical elastic differential cross sections for 715 MeV/c $K^{+}+{}^{12}$C as a function of center of mass angle. The results obtained using the KDP-RIA relativistic optical model are plotted as a band of values which take into account the $\pm$ 15% uncertainty in the elementary $K^{+}N$ amplitudes.



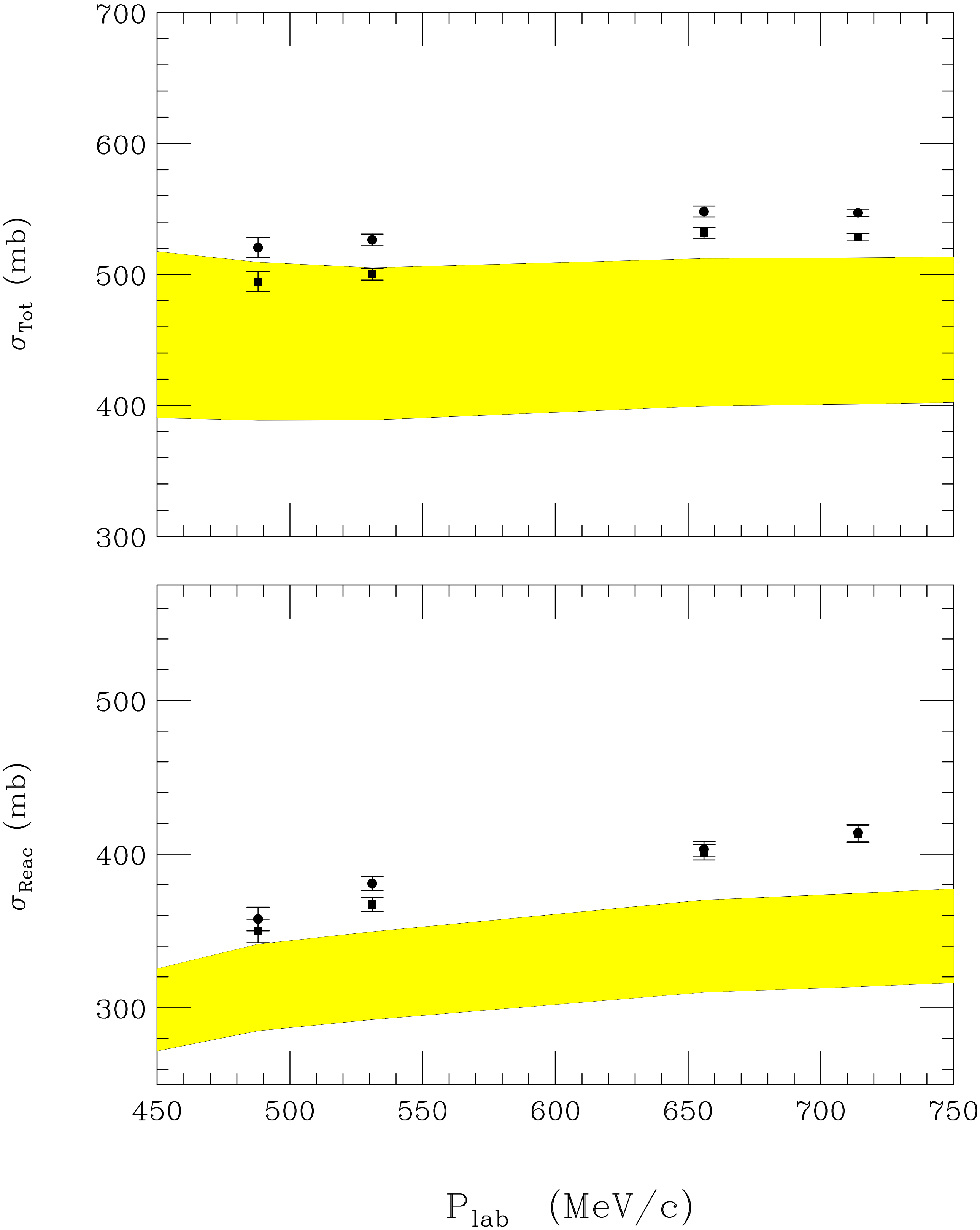
100
90
80
70
60
50
σ_{Tot} (mb)
σ_{Reac} (mb)
450
500
550
600
650
700
750
P_{lab} (MeV/c)



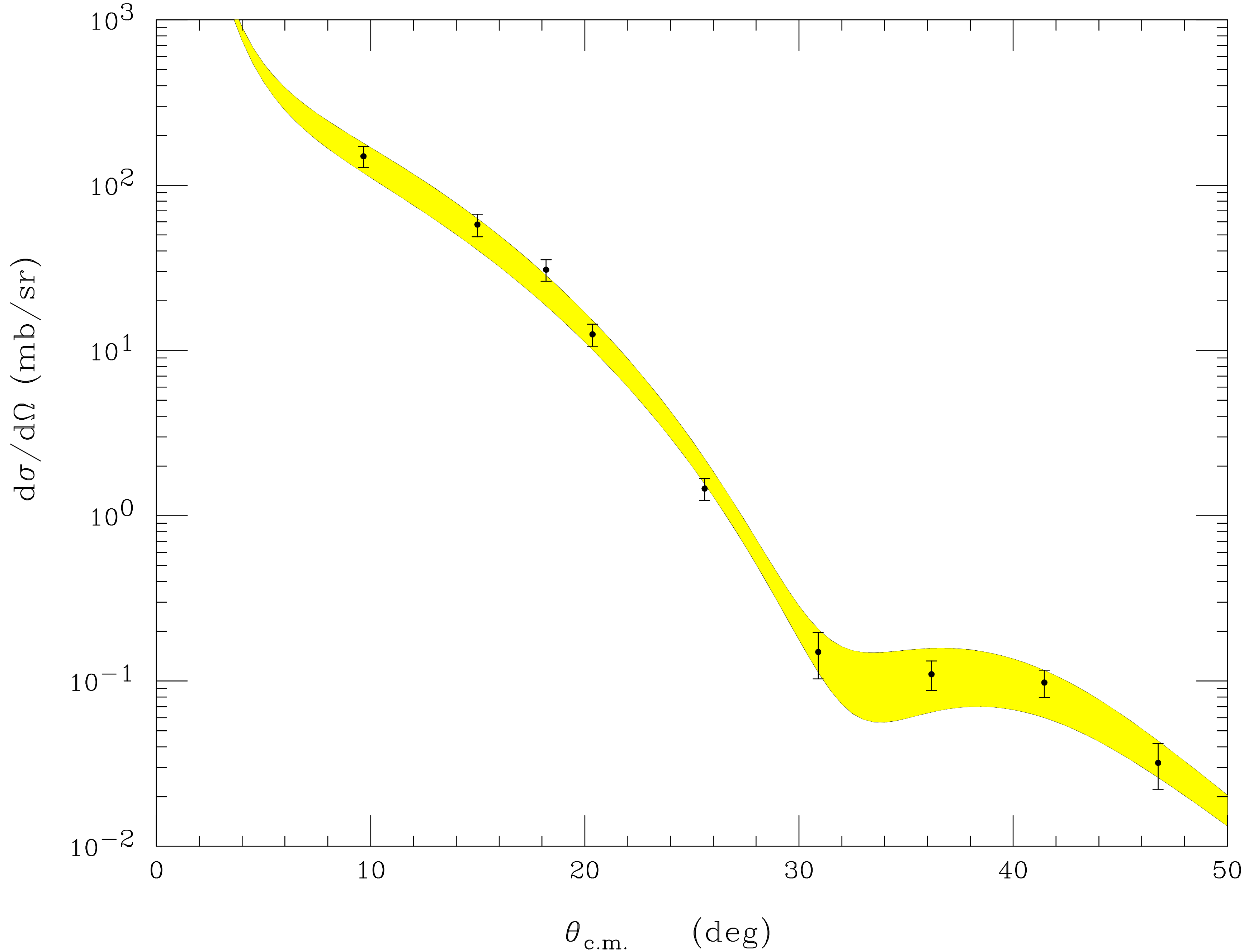
σ_Tot (mb)
200
180
160
140
120
100
σ_Reac (mb)
180
160
140
120
100
80
450
500
550
600
650
700
750
P_lab (MeV/c)



450
400
350
300
250
σ_{Tot} (mb)
400
350
300
250
200
σ_{Reac} (mb)
450
500
550
600
650
700
750
P_{lab} (MeV/c)



700
600
500
400
300
σ_{Tot} (mb)
500
400
300
σ_{Reac} (mb)
450
500
550
600
650
700
750
P_{lab} (MeV/c)



$d\sigma/d\Omega$ (mb/sr)
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
0
10
20
30
40
50
$\theta_{c.m.}$ (deg)